Key to "OEIS.org 2014" Poster

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December 14, 2014

The poster[1] "OEIS.org 2014" shows nine especially interesting sequence in the *On-Line Encyclopedia of Integer Sequences* or *OEIS* (http://oeis.org) that have been studied recently. It commemorates the OEIS reaching a quarter of a million sequences, and will be distributed at the 2015 Joint Mathematics Meetings in San Antonio, Texas.

An essential requirement for inclusion in the poster was a noteworthy illustration. Furthermore, all except S_5 and S_8 represent essentially unsolved problems. For further information about any of these sequences, see the corresponding entry in the OEIS. For example, **A250000** can be found at the URL http://oeis.org/A250000.

Denote the nine sequences illustrated in the poster by

$$S_1,\ S_2,\ S_3,$$
$$S_4,\ S_5,\ S_6,$$
$$S_7,\ S_8,\ S_9.$$

As in the OEIS, $a(n)$ denotes the nth term of the sequence being discussed.

S_1: **A250001: Number of ways to arrange n circles in the plane.** Two circles are either disjoint or meet in two points, and two circles may not be tangent to each other. Three circles may not meet at a point. Two arrangements are considered the same if one can be continuously changed to the other while keeping all circles circular (although the radii may be continuously changed), without changing the multiplicity of intersection points, and without a circle passing through an intersection point. Turning the whole configuration over is allowed. How many different arrangements are possible?

Jonathan Wild found the only terms that are known: $a(1) = 1$, $a(2) = 3$, $a(3) = 14$, $a(4) = 168$. The figure shows nine of the 14 arrangements of three circles.

If we allow circles to be tangential, or allow more than two circles to meet at a point, nothing is known.

[1] It can be freely downloaded from the OEIS Foundation's web site, http://oeisf.org, together with this "Key".

For the number of ways to arrange n lines in the (affine) plane, rather than circles, see **A241600**. Here seven terms are known: 1, 2, 4, 9, 47, 791, 37830.

S_2: **A250000: Peaceable coexisting armies of queens.** The maximum number m such that m white queens and m black queens can coexist on an $n \times n$ chessboard without attacking each other. Only the values $a(1), \ldots, a(13)$ are known: 0, 0, 1, 2, 4, 5, 7, 9, 12, 14, 17, 21, 24 (for references etc., see the entry). The figure shows $a(11) = 17$ white queens and 17 black queens on an 11×11 board, with no queen of one color attacking any queen of the opposite color. This sequence, an interesting combinatorial problem with a peaceful theme, was the winner of the competition to decide which entry would be numbered A250000, in celebration of reaching a quarter of a million sequences and the 50th anniversary of the creation of the database. Sequence S_1 was the runner-up.

S_3: **A160860: Least number of pieces from cutting a convex n-gon along all its diagonals.** Only the values $a(3), \ldots, a(8)$ are known: 1, 4, 11, 24, 47, 80, although there are conjectures for $a(9)$ and $a(2t)$ (V. Letsko et al.). The figure shows how to draw a convex heptagon so that cutting along all its diagonals produces the minimum number, $a(7) = 47$, of pieces. The figure also illustrates **A230281**(7)=29 (minimizing the number of intersection points).

S_4: **A250120: Coordination sequence of planar net $\mathbf{3^4.6}$**. Start at a node, and count the nodes that are n edges away. For the eleven uniform (or Archimedean) planar nets, it does not matter at which node one starts. The figure shows the coordination sequence for the $3^4.6$ net, A250120:

$$1,\ 5,\ 9,\ 15,\ 19,\ 24,\ 29,\ 33,\ 39,\ 43,\ 48,\ 53,\ 57,\ \ldots$$

(there is a conjectured recurrence). We start at the central black node, which is surrounded by 5 red nodes one edge away, then 9 green nodes at distance 2, 15 yellow nodes at distance 3, and so on. The figure was drawn by D. Chavey.

The coordination sequences for the other uniform planar nets are **A008458** (3^6), **A008486** (6^3), **A008574** (remarkably, both 4^4 and 3.4.6.4), **A008576** (4.8^2), **A008579** (3.6.3.6), **A008706** ($3^3.4^2$), **A072154** (4.6.12), **A219529** (3^2.4.3.4), **A250122** (3.12^2).

Many more coordination sequences need to added to the OEIS. For example, each of the 20 2-uniform planar tilings (cf. **A068599**) will give rise to a pair of sequences - see B. Grünbaum and G. C. Shephard, *Tilings and Patterns*, Freeman, New York, 1987, pp. 66-67.

S_5: **A160239: Number of ON cells after n generations of the cellular automaton known as "Fredkin's Replicator".** I will be talking about this at 09:30-10:00 on Sunday, Jan 11 in the Special Session on *Enumerative Combinatorics*. A manuscript is in preparation.

S_6: **A227133: Maximum number of points in an $n \times n$ grid such that there is no square.** We only care about squares whose sides are parallel to the axes. Just ten terms $a(1), \ldots, a(10)$ are known: 1, 3, 7, 12, 17, 24, 32, 41, 51, 61 (H. Ludwig, G. Resta). The figure shows the maximum number, $a(8) = 41$, of red points in an 8×8 grid such that no four of them form a square.

S_7: **A232467: Corner designs.** The figure shows a $(20, 2)$ corner design of C. S. Kaplan, "redrawn from the menu of *Os Tibetanos*, a Tibetan restaurant in Lisbon, Portugal". The reader is referred to Kaplan's fascinating paper (*Bridges, 2013*) for the definition of corner designs and the lovely—and apparently unsolved—problem of their enumeration. See A232467 for further information.

S_8: **A230628: Colors needed for a map of empires.** In the four-color problem, two adjacent countries must have different colors, and at most four colors are needed. Here we have "empires", where each empire consists of n disjoint countries, and two adjacent countries from different empires must have different colors. In the worst case, how many colors are needed? The answer is known, from the work of B. Jackson and G. Ringel: $a(1) = 4$ (this is the four-color theorem), and $a(n) = 6n$ for $n \geq 2$. The illustration (from I. Stewart) shows a map of twelve empires of size 2 requiring $a(2) = 12$ colors.

S_9: **A229037: Greedy sequence with no 3-term arithmetic progression.** This is the sequence of positive integers where each is chosen to be as small as possible subject to the condition that no three terms $a(j)$, $a(j + k)$, $a(j + 2k)$ (for any j and k) form an arithmetic progression:

$$1,\ 1,\ 2,\ 1,\ 1,\ 2,\ 2,\ 4,\ 4,\ 1,\ 1,\ 2,\ 1,\ 1,\ 2,\ 2,\ 4,\ 4,\ 2,\ 4,\ 4,\ 5,\ 5,\ 8,\ 5,\ \ldots$$

Submitted by J. W. Grahl, with 10000 terms computed by G. Resta, A P. Heinz, and C. R. Greathouse, with the amazing graph shown in the figure. *What* is going on?